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THE THEOREM OF MOMENTS.

By DR. C. G. KNOTT, University of Edinburgh.

Professor Huntington's recent communication on the "Theorem of Moments" (in the AMERICAN MATHEMATICAL MONTHLY for December, 1914) calls attention to a point of real importance. He designedly confined his discussion to the elementary case of uniplanar rigid dynamics; but it struck me that the extraordinarily simple proof of the general case in terms of quaternion analysis might be of interest to readers of the MONTHLY.

I assume Newton's second and third laws of motion or their equivalents.

Let the mass m in position ρ be acted on by the force β. Then for one particle[1]

$$m\ddot{\rho} = \beta$$

and for a system of particles

$$\Sigma m\ddot{\rho} = \Sigma\beta \tag{1}$$

in which $\Sigma\beta$ includes only the applied or external forces (Lex III).

Also, introducing each corresponding ρ as multiplier, and summing for the system, we have

$$\Sigma m\rho\ddot{\rho} = \Sigma\rho\beta. \tag{2}$$

This breaks up into the scalar and vector parts

$$\Sigma Sm\rho\ddot{\rho} = \Sigma S\rho\beta$$

and

$$\Sigma Vm\rho\ddot{\rho} = \Sigma V\rho\beta. \tag{3}$$

The first of these (2) is the "virial" equation and was first used by Hamilton.

The latter equation (3) expresses the theorem of moments; and in it also, by virtue of Lex III, $\Sigma V\rho\beta$ includes only the applied forces.

Now let π be the vector position of any point at rest or in motion, and let σ be the vector position with respect to this new point of any element of the system. Then

$$\rho = \pi + \sigma, \qquad \ddot{\rho} = \ddot{\pi} + \ddot{\sigma}.$$

Equations (1) and (2) become

$$\Sigma m(\ddot{\pi} + \ddot{\sigma}) = \Sigma\beta, \tag{1'}$$

$$\Sigma m(\pi + \sigma)(\ddot{\pi} + \ddot{\sigma}) = \Sigma(\pi + \sigma)\beta. \tag{2'}$$

Since π is the vector position of a definite point, it may be taken outside or placed within the summation symbol at will. Multiplying π into (1′) and subtracting from (2′) we find

$$\Sigma m\sigma(\ddot{\pi} + \ddot{\sigma}) = \Sigma\sigma\beta$$

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[1] See my edition (1904) of Kelland & Tait's *Introduction to Quaternions*, page 160.

and the vector part of this is

$$V(\Sigma m\sigma)\ddot{\pi} + \Sigma Vm\sigma\ddot{\sigma} = \Sigma V\sigma\beta. \tag{4}$$

Now if the point π is to be a point with regard to which we may apply the theorem of moments in the form (3), we must have

$$\Sigma Vm\sigma\ddot{\sigma} = \Sigma V\sigma\beta \tag{5}$$

and hence the condition to be satisfied by π is simply

$$V(\Sigma m\sigma)\ddot{\pi} = 0. \tag{6}$$

But, in general, $\Sigma m\sigma = \mu\Sigma m$, where μ is the vector position of the center of mass of the system referred to the point π. Hence (6) becomes

$$V\mu\ddot{\pi} = 0. \tag{7}$$

This equation may be satisfied by any one of the conditions

$$(1)\ \mu = 0; \qquad (2)\ \ddot{\pi} = 0; \qquad (3)\ \ddot{\pi} \text{ parallel to } \mu.$$

Hence we may legitimately use the point whose vector is π as a pivot for moments when

(1) it is the center of mass; or

(2) it has no acceleration so that its velocity is constant (the case of) rest or uniform motion in a straight line; or

(3) its acceleration is directed towards the center of mass.

Since the acceleration of the center of mass is necessarily directed through its own position, condition (3) really includes (1). And thus, quite generally, the theorem of moments may be applied with reference to any point whose acceleration is either zero or directed through the center of mass.

BOOK REVIEWS

MATHEMATICS IN DR. ELIOT'S FIVE-FOOT SHELF OF BOOKS.

By W. H. BUSSEY, University of Minnesota.

Several years ago the attention of the whole country was attracted by the announcement made by Dr. Charles W. Eliot that he proposed to select a group of books, filling a five-foot shelf, in the belief that "the faithful and considerate reading of these books, with such re-readings and memorizings as individual taste may prescribe, will give any man the essentials of a liberal education, even if he can devote to them but fifteen minutes a day." The selection was made by him in due time, and the books have since been published by P. F. Collier and Son in a fifty volume set called "The Harvard Classics." In the introduction